Avoiding Being Trapped in False Analogical Modeling of Composite Wall Thermal Resistance

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Because Analogy is considered as a double-edged sword, thermal engineers should be cautious in analogical maneuvering between electrical and thermal domains in order not to be slipped into building misconceptions about thermal resistance concept. Composite wall thermal resistance (CWTR) modeling is one of the practical examples that illustrates the probability of misusing analogy. Heat transfer undergraduate textbooks coverage of CWTR suffers a lean towards "cookbook" coverage that reports concise statements that lack deep clarification and illustration. Transparent Thinking Approach (TTA) is employed to present a detailed calculation and illustration of a typical CWTR modeling based on isothermal and adiabatic assumptions. The calculation of a typical CWTR for different values of wall thermal conductivities shows that the difference in parallel walls thermal conductivity is creating a large discrepancy that may reach 80% between heat flows calculated based on isothermal and adiabatic assumptions. It is found that for a series-parallel arrangement of composite walls with high difference in parallel wall thermal conductivity values, the true value of heat flow is bracketed between the isothermal and adiabatic heat flow values. The transparent way of presenting CWTR modeling can be readily included in any standard heat transfer textbook and result in greatly enhancing CWTR modeling coverage.

Keywords**:** Heat Conduction; Thermal Resistance Modeling; False Analogy; Composite Wall; Transparent Modeling Approach

Introduction

Einstein and Infeld (1967) in their book "the evolution of physics" *described deeply* the important role of analogy between phenomena that seams distant and how these phenomena look after being *deeply and analogically analyzed*.

> "*It has often happened in physics that an essential advance was achieved by carrying out a consistent analogy between apparently unrelated phenomena….The association of solved problems with those unsolved may throw new light on our difficulties by suggesting new ideas. It is easy to find a superficial analogy which really expresses nothing. But to discover some essential common features, hidden beneath a surface of external differences, to form, on this basis, a new successful theory, is important creative work.*" *Einstein and Infeld (1967)*

This quote indicates the important role that analogy plays in *deepening* our understanding of the surrounding phenomena and ending in discovering useful connections. In other words, we try to reveal the *hidden connections* inside things or phenomena in order to deeply understand them. Analogy is one of the most important cognitive tools that helps in creating *deep and meaningful learning*. These deep insights are sometimes hidden under *superficial analogy*. Analogy (and similarly metaphor and simile) is considered a cognitive device that creates a relationship between things, processes, or concepts in an *analog domain* with similar ones in a *target domain*. Learners are *intuitively drifted* to employ what they already fully understand in a certain analog domain to structure a similar *deep understanding* in another target domain.

A *highway of transfer of knowledge* between the two domains is established which results in more elaborated connections, namely deeper understanding. The product of the deeper understanding is a transferable knowledge between the two domains (Jonane, 2015). *Meaningful learning* occurs when the learners visualize connections between newly studied material and what they already know. Analogy mapping between the analog domain and target domain to create connections and insights is a form of meaningful learning. As the learner dig deeper in creating connections between domains, a more meaning is created. Links created by analogy promote conceptual understanding and create coherence between prior knowledge and newly structured one (Harrison, 2006).

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1 Materials and Methods

1.1 Avoiding being drifted into misunderstanding zone

Analogy is a device that connects two entities that are similar in certain aspects, but they are also different in others. Learners and instructors have to be careful in using this cognitive device in order not to be drifted into the misunderstanding zone. Glynn (2008) wrote about this:

> *"analogies are double-edged swords: They can foster understanding, but they can also lead to misconceptions".* As Duit, *et al*. (2001) explains:
>
> *"A growing body of research shows that analogies may be powerful tools for guiding students from their pre-instructional conceptions towards science concepts. But it has also become apparent that analogies may deeply mislead students' learning processes. Conceptual change, to put it into other words, may be both supported and hampered by the same analogy" Duit, et al. (2001)*

Learners and teachers should keep in their minds that analogy usually leads to meaningful and deep learning if it is *used carefully*. But, if analogy is *misused*, it may result in forming misconceptions.

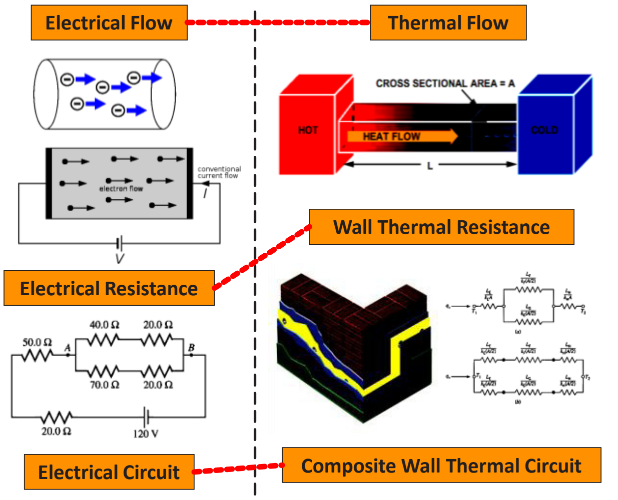

Fig. 1 Thermal-Electric Analogy of resistances in series-parallel arrangement

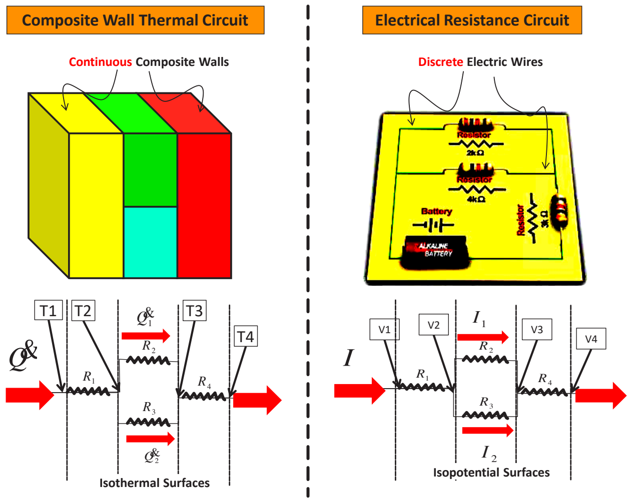

Fig. 2 Comparing and Contrasting Composite wall circuit with electrical resistance circuit

1.2 Thermo-Electric analogy in modeling CWTR

Thermal-electrical analogy is still an important model that is implemented in modeling energy transfer processes (Capizz, *et al*., 2017, Haoa, *et al*., 2018, and Roslan, *et al*., 2017). The concept of thermal resistance is deeply engrained in the knowledge structure of thermal engineers (Lasance, 2008). As illustrated in **Figure 1**, an electrical-thermal analogy is constructed to result in intuitively feeling that as electricity flow by voltage (driving force) and confronted by electrical resistance, thermal energy also flows under temperature difference (driving force) and faced by thermal resistance. As an electrical resistance circuit is formed by a certain arrangement of resistances, also thermal circuits similarly can be formed by a certain arrangement of thermal resistances. Steady-state thermal analysis of composite walls is one of the most prominent applications for the use of thermal resistance network analysis, as shown in Fig. 1.

Analogical modeling is to be employed in this paper to clarify where we should be careful when dealing with electrical-thermal analogy and how to avoid being slipped into the misunderstanding zone. In the following sections, an illustrative calculation for the thermal resistance of a typical composite wall based on appropriate assumptions will be presented to show how analogy can be safely implemented.

1.3 Analogical Modeling of CWTR

1.3.1 Avoiding blind implementation of thermo-electric analogy

As stressed before, analogy is a very useful cognitive devise, but it should be used wisely in order not to be "slipped into" false analogy. As explained before, the analogy between heat flow by conduction (Fourier's Law) and the electrical current (Ohm's Law) is correct in certain aspects, but they are totally different in others. The corresponding properties of thermal conductivity and electrical conductivity are practically different in nature which makes heat flow and current flow behaves differently. As illustrated in Fig. 1, heat and current flow are similar but are also physically different from the following aspects (Lasance, 2008). Electric current flows in wires (Discrete Channels) with no dissipation of current outside these wires, but thermal energy flows in walls (Continuous Channels) with possibilities to be dissipated in all directions. Therefore, it is hard to guarantee adiabatic conditions in thermal flow inside continuous walls, as illustrated in **Figure 2**.

The electrical resistance is defined as the difference in potential divided by current flowing in a wire between two points. This definition is based on an underlying assumption of equipotential (or can be called isopotential in analogy with isothermal) surfaces, as illustrated in Fig. 2. This assumption is practically correct for electrical circuits, but it is not completely correct in thermal flow to assume isothermal surfaces while there is no guarantee of any heat dissipation in walls in all directions.
The range of electrical conductivity values between an electrical insulator and an electrical conductor is about 20 orders of magnitude, but it is only three orders of magnitude between thermal "insulator" and "conductor", as illustrated in **Figure 3**. From a practical point of view, electric current in wires is one dimensional, but the heat flow is inherently a 3D phenomenon that can be approximated by a 1D scenario. Thermal flow walls cannot be perfect insulators and heat lost by radiation occurs even in a vacuum and it is impossible to get losses to zero.

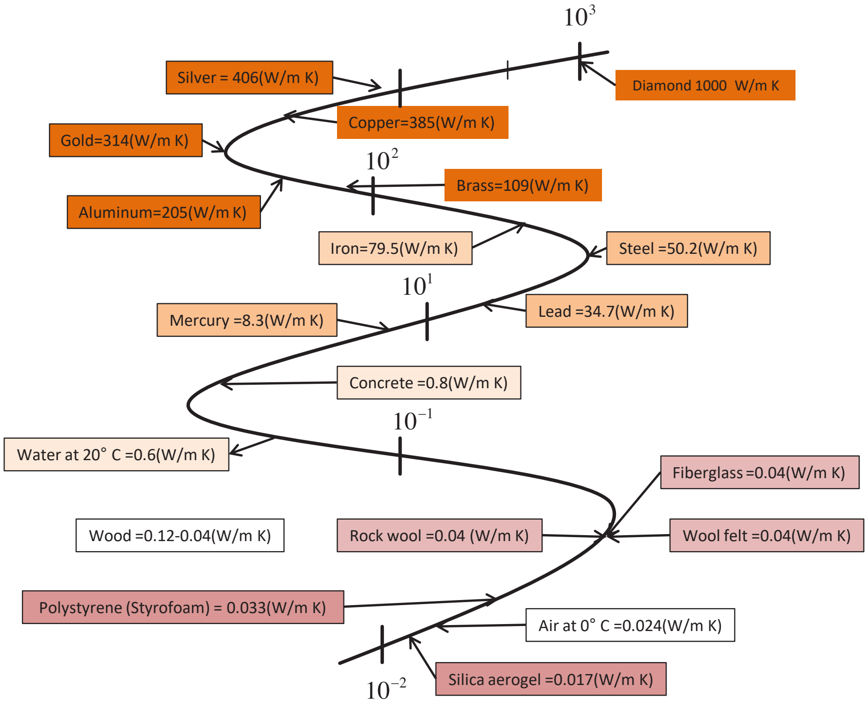

Fig. 3 The whole spectrum of thermal conductivity values

1.4 CWTR presentation in heat transfer textbooks

Based on my long experience of teaching undergraduate Chemical Engineering courses, it can be concluded that undergraduate textbook content material should not be presented in a "cookbook" style and the text should clearly highlight the fundamental and basic concept understanding by showing the assumptions used and its *validity, justifications, and limitations*. Adopting the "cookbook" presentation style without getting deep into the understanding of basic assumptions and limitations send a wrong message to students that the material are not studied to reinforce basic concepts that can be extended to real-life problems, but it is studied to solve problems similar to examples and end-chapter problems. Also, based on my long experience of teaching the Heat Transfer Course for undergraduate students, it is found that heat transfer textbooks presentation for the calculation of thermal resistance for parallel-series arrangements of composite walls needs a *lot of improvement*. Improvement is needed because this topic is based on thermo-electric analogy and, as it is mentioned before, the analogy should be carefully dealt with in order not to be slipped in building misconceptions. Cengel, 2003, wrote in his textbook entitled "Heat Transfer: Practical Approach":

> *"The R-value of a wall or roof structure that involves layers of uniform thickness is determined easily by simply adding up the unit thermal resistance of the layers that are in series. But when a structure involves components such as wood studs and metal connectors, then the thermal resistance network involves parallel connections and possible two-dimensional effects. The overall R-value, in this case, can be determined by assuming (1) parallel heat flow paths through areas of different construction or (2) isothermal planes normal to the direction of heat transfer. The first approach usually over predicts the overall thermal resistance, whereas the second approach usually predicts it. The parallel heat flow path approach is more suitable for wood-frame walls and roofs, whereas the isothermal planes approach is more suitable for masonry or metal frame walls." (Cengel, 2003)*

These statements are considered vague concepts to students unless it is supported by graphical illustrations and detailed calculations as this paper will suggests. Undergraduate students are usually not familiar with heat transfer concepts and cannot understand deeply isothermal and adiabatic assumptions unless it is presented clearly using graphical aids and detailed sample calculations. Incropera, *et al*., (2002), also wrote in their book entitled “Fundamentals of Heat and Mass Transfer”:

> *“Composite walls may also be characterized by series-parallel configurations, such as that shown in Figure 3.3. Although the heat flow is now multidimensional, it is often reasonable to assume one-dimensional conditions. Subject to this assumption, two different thermal circuits may be used. For case (a) it is presumed that surfaces normal to the x-direction are isothermal, while for case (b) it is assumed that surfaces parallel to the x-direction are adiabatic. Different results are obtained for Rtot and the corresponding values of q bracket the actual heat transfer rate. These differences increase with increasing* k_F-k_G*, as multidimensional effects become more significant” (Incropera et al., (2002))*

These statements are not enough for undergraduate students to deeply understand these two important assumptions that the thermal circuits that are built based on them unless the textbook is enhanced by effective graphical illustrations and detailed sample calculations. Ozisik, (1980), and Holman, (1986), wrote even shorter statements about these two-important adiabatic and isothermal assumptions. Based on heat transfer textbook thermal network converge review, textbooks’ presentation of thermal resistance calculations should be improved by elaboration on the deep differences that exist between thermal and electric analogy, the deep understanding of isothermal and adiabatic assumptions, detailed numerical examples that shows the adequacy of each assumption, and graphically enhance the intuitive students feeling of one dimensional and multidimensional heat flow.

1.5 Transparent analogical modeling tool

While reading throughout this paper you may be surprised how the author is graphically and analogically modeling the content knowledge. This enhanced way of presentation is one of the *fruits* of implementing the author’s newly developed thinking approach that is called “Transparent Thinking Approach (TTA)” (Aliedeh, 2015a, b, and c). In TTA, modeling is considered as a representation process of what the transparent thinker perceives from maneuvering between perspectives. Maneuvering between perspectives crucially needs modeling tools to record these shots and connect them together in a visible model. TTA looks to modeling from the widest perspective. Transparent modeling gathers a wide spectrum of tools such as pictures, physical models, analogy, metaphor, conceptual frameworks, drama, video, storytelling, journey, mathematics, Language, fashion model, role model ….. etc., as illustrated in **Figure 4** (Aliedeh, 2016, 2017 and 2018). All these are some in a long list of tools that learners should use to record their understanding.

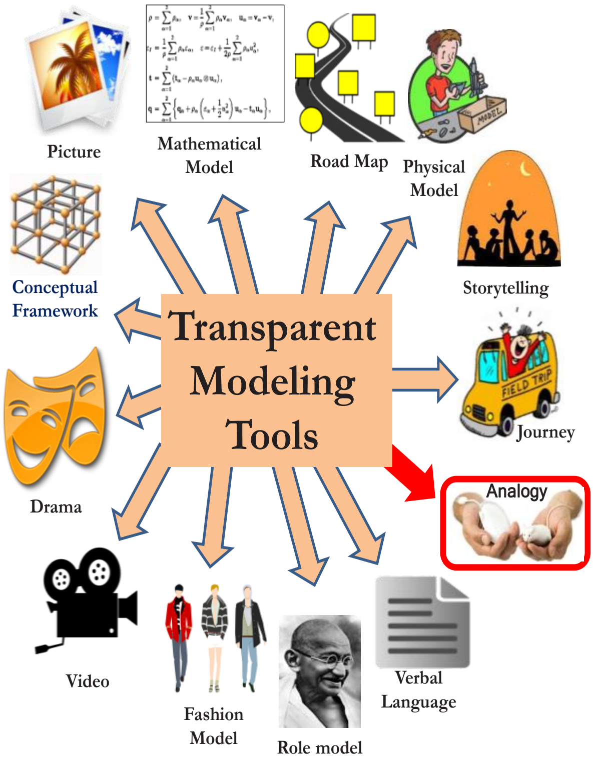

Fig. 4 Transparent Modeling Tools (Aliedeh, 2015 a, b, c, 2016, 2017 and 2018)

2 Results and Discussion

2.1 Transparent Modeling and Calculations Of CWTR

2.1.1 A typical example of CWTR modeling

Thermal flow in composite wall is *inherently a three-dimensional flow phenomenon* because composite walls are continuous medium and there are no barriers inside the wall that hinder the movement of heat in all directions. Even if there is no guarantee that the flow is going to be restricted to one direction, the thermal flow in composite walls can be *approximated* by assuming it as *one-dimensional flow*. A typical composite wall, shown in **Figure 5**, is taken as an example to show that thermal engineers should not consider blindly composite wall thermal resistance circuits exactly similar to electrical circuits without paying attention to the fact that the nature of heat flow in composite wall is different than the nature of electrical circuits. This blind analogy between thermal and electrical circuits will result in a high discrepancy between total composite walls thermal resistance calculated values.

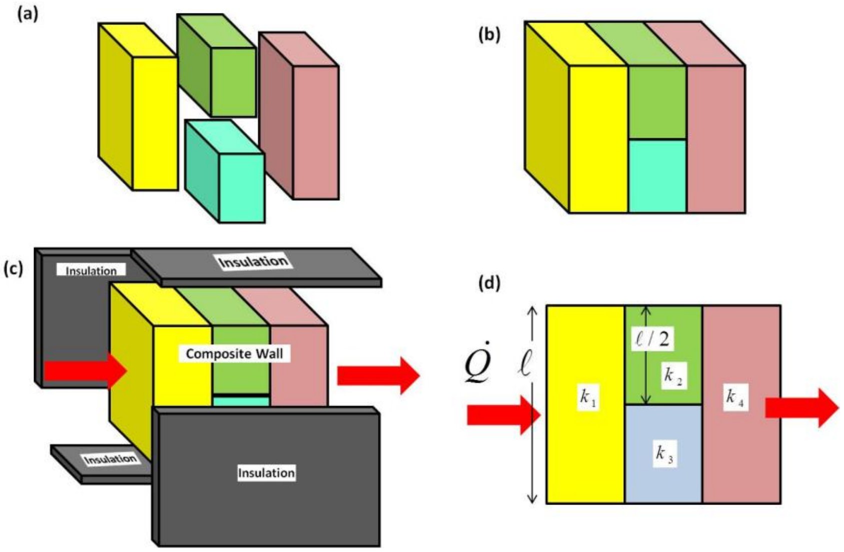

Fig. 5 A typical wall Composite of four different materials in series-parallel arrangement

As illustrated in Fig. 5, a typical wall is constructed from four blocks of different materials. To calculate total thermal resistance and heat flow rate that passes through this composite wall, two important assumptions are needed to be imposed at this point: Steady-state Assumption: This is met by *controlling the input variables* and giving the system *enough time* to reach steady-state conditions. This condition can be easily and practically applied. One-dimensional thermal flow assumption: The walls arrangement is insulated from all directions except from right and lift sides in order to help in directing the thermal flow in one direction.

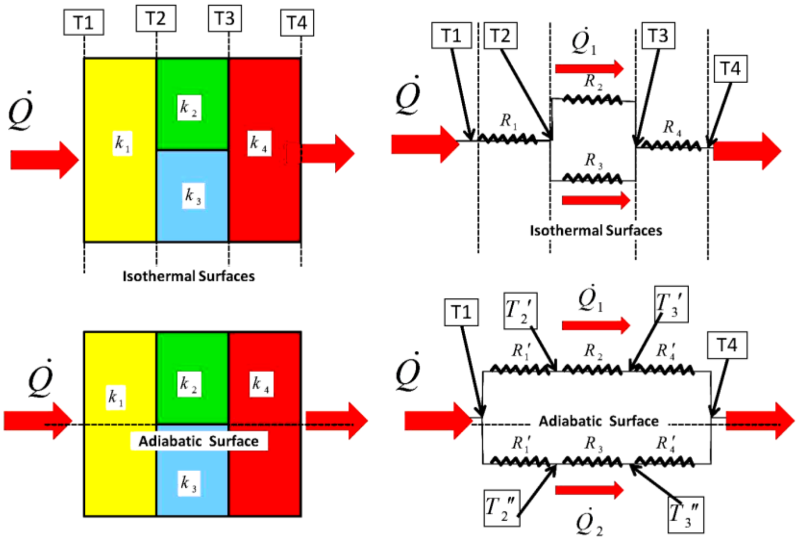

Fig. 6 Comparing adiabatic and isothermal surfaces in a typical composite walls arrangement

2.1.2. Actual conditions inside the composite wall (Isothermal vs. Adiabatic)

When deeply analyzing how much the 2nd assumption of one-dimensional flow is close to reality, a question arises about what the actual conditions inside the composite wall is, as illustrated in **Figure 6**. Can we assume an isothermal condition to be valid and deal with the thermal circuits exactly as we usually deal with electrical circuits? The isothermal thermal condition *contradicts the one-dimensional assumption* because isothermal surfaces are created by allowing heat flow to move in more than one direction. If adiabatic conditions are assumed to be valid which compatible with the one-dimensional assumption but this condition *contradicts the common-sense feeling* that adiabatic conditions cannot be fully guaranteed in composite walls. So the truth lies between these two assumptions as the following detailed calculations of thermal resistance of the typical composite wall that will be shown in the following sections based on the above two assumptions, as illustrated in Fig. 6.

2.1.3 Total thermal resistance calculation based on isothermal assumption

The typical composite wall, shown in Fig. 5 (d), is analyzed based on isothermal assumption as shown in **Figure 7** (a). The following are a step-by-step total resistance calculation for the composite wall and it is graphically illustrated in Fig. 7 (b).

$$R_1=\frac{\Delta x}{k_1(A)},R_2=\frac{\Delta x}{k_2(A/2)}=\frac{2\Delta x}{k_2(A)},R_3=\frac{\Delta x}{k_3(A/2)}=\frac{2\Delta x}{k_3(A)},R_4=\frac{\Delta x}{k_4(A)}$$

$$R'=\frac{1}{\dfrac{1}{\dfrac{2\Delta x}{k_2(A)}}+\dfrac{1}{\dfrac{2\Delta x}{k_3(A)}}}=\frac{1}{\dfrac{k_2(A)}{2\Delta x}+\dfrac{k_3(A)}{2\Delta x}}=\frac{2\Delta x}{A\left\{k_2+k_3\right\}}$$

$$R_{tot-Isothermal}=\frac{\Delta x}{A}\left\{\frac{1}{k_1}+\frac{2}{\left\{k_2+k_3\right\}}+\frac{1}{k_4}\right\} \tag{1}$$

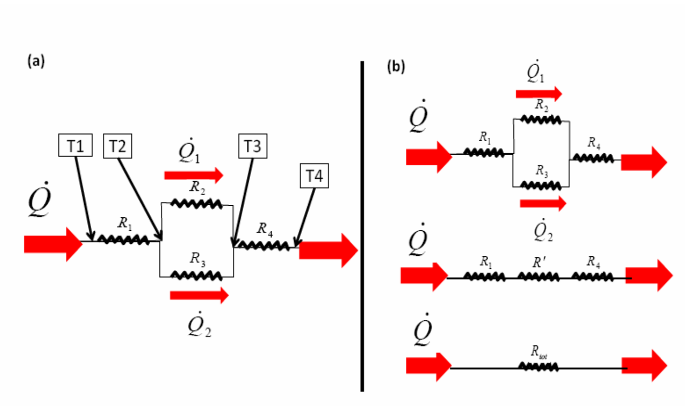

Fig. 7 Steady-state one-dimensional thermal resistance circuit representation based on vertical isothermal surfaces assumption

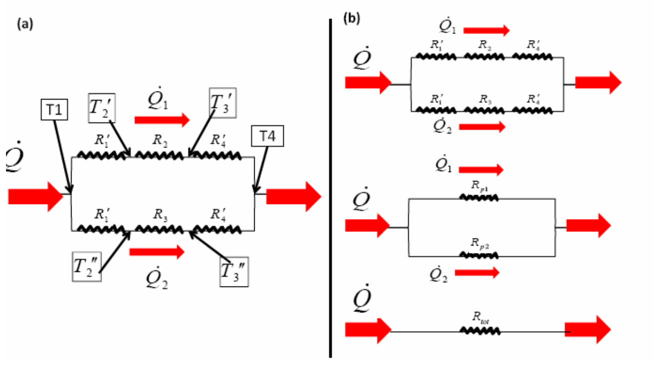

Fig. 8 Steady-state one-dimensional thermal resistance circuit representation based on horizontal adiabatic assumption

2.1.3Total thermal resistance calculation based on adiabatic assumption

The typical composite wall, shown in Fig. 5 (d), is analyzed based on isothermal assumption as shown in **Figure 8**. (a). The following equation are a step-by-step total resistance calculation for the composite wall and its graphically illustrated in Fig. 8 (b).

$$R_1'=\frac{2\Delta x}{Ak_1},R_2=\frac{\Delta x}{k_2(A/2)}=\frac{2\Delta x}{k_2(A)},R_3=\frac{\Delta x}{k_3(A/2)}=\frac{2\Delta x}{k_3(A)},R_4'=\frac{2\Delta x}{Ak_4}$$

$$R_{p1}=\frac{2\Delta x}{Ak_1}+\frac{2\Delta x}{Ak_2}+\frac{2\Delta x}{Ak_4}=\frac{2\Delta x}{A}\left\{\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_4}\right\}$$

$$R_{p1}=\frac{2\Delta x}{Ak_1}+\frac{2\Delta x}{Ak_3}+\frac{2\Delta x}{Ak_4}=\frac{2\Delta x}{A}\left\{\frac{1}{k_1}+\frac{1}{k_3}+\frac{1}{k_4}\right\}$$

$$R_{tot-Adiabatic}=\frac{2\Delta x}{A}\frac{1}{\dfrac{1}{\left\{\dfrac{1}{k_1}+\dfrac{1}{k_2}+\dfrac{1}{k_4}\right\}}+\dfrac{1}{\left\{\dfrac{1}{k_1}+\dfrac{1}{k_3}+\dfrac{1}{k_4}\right\}}} \tag{2}$$

2.2 Discrepancy between isothermal and adiabatic assumptions

The two equations derived above (Equations 1 and 2) are used to calculate the total composite wall thermal resistance based on isothermal and adiabatic conditions, as listed in **Table 1**. The calculation is done for a numerical value listed in Table 1 for 12 different cases using Mathcad Software. In each case, the value of a certain input parameter is changed while the others are kept at the same value of the reference values in the first case. For each case, the value of thermal flow rate is calculated for isothermal and adiabatic conditions and then the percent discrepancy between them is also calculated based on equation (3):

$$\left\{\begin{matrix}\text{\% Disrepancy in the}\\ \text{calculated Thermal Flowrate}\end{matrix}\right\}=\frac{\dot{Q}_{\text{Isothermal}}-\dot{Q}_{\text{Adiabatic}}}{\dot{Q}_{\text{Adiabatic}}}\times 100\% \tag{3}$$

Table 1 Calculating the percent discrepancy between thermal rates based on isothermal and adiabatic assumptions arrangements

Case #	$W/m\cdot C$				$\Delta x\ (m)$	$A\ (m^2)$	$\Delta T\ (K)$	$\dot{Q}\ (W)$		% Discrepancy
	k_1	k_2	k_3	k_4				Isothermal	Adiabatic	
1	10	20	30	40	0.1	1	30	1818	1805	0.752
2	10	20	30	40	0.01	1	30	18180	18050	0.758
3	10	20	30	40	0.1	10	30	18180	18050	0.758
4	10	20	30	40	0.1	1	100	6061	6015	0.758
5	100	20	30	40	0.1	1	30	4000	3960	1.014
6	10	20	30	100	0.1	1	30	2000	1984	0.806
7	10	20	10	40	0.1	1	30	1565	1524	2.717
8	10	20	1	40	0.1	1	30	1362	990.4	37.52
9	10	20	0.1	40	0.1	1	30	1336	871.9	53.2
10	10	20	0.01	40	0.1	1	30	1334	858.6	55.3
11	10	100	0.01	40	0.1	1	30	2069	1113	85.9
12	10	20	20	40	0.1	1	30	1714	1714	0

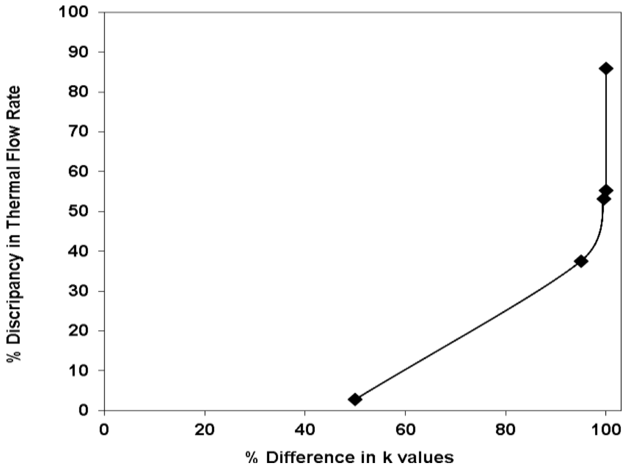

Fig. 9 The effect of the difference in parallel *k* value in increasing the discrepancy in thermal flow rates based on isothermal and adiabatic assumptions

As listed in Table 1, It is noticed that the only change that affect significantly the discrepancy between isothermal and adiabatic calculated thermal flow rate is making a significant difference between the values of conductivity of the walls in parallel (k_2 and k_3) (Cases 7 to 12 in Table 1). To illustrate this finding, the % difference in the parallel k values is calculated based on equation (4):

$$\left\{\begin{matrix}\text{\% Difference in the}\\ \text{Parallel k values}\end{matrix}\right\}=\frac{k_2-k_3}{k_2}\times 100\% \tag{4}$$

Percent difference in parallel *k* values is calculated for cases 7-12. **Figure 9** shows the relationship between % difference parallel conductivity and the discrepancy in thermal flow values. Fig. 9 illustrates how the difference in parallel *k* values is the major factor in affecting a big discrepancy between isothermal and adiabatic arrangements. The discrepancy reaches zero when the same value is assigned to parallel k values, namely series arrangement (Cases 12 in Table 1). This discrepancy sours high and reaches more than 80% when there is about 100% difference in parallel k values, as shown in Fig. 9. Table 1, and Fig. 9 stressed the fact that the CWTR model is not only represented by isothermal nor by adiabatic, and these two assumptions are bracketing the true value of thermal flow, as illustrated by

Figure 10. When the values of the parallel walls conductivities approach each other as in case 12, Table 1, the discrepancy reaches zero. Therefore, series thermal circuits and electrical circuits are equivalent. When parallel-series arrangements is built with big differences in parallel walls conductivities, the deviation between isothermal and adiabatic model starts to be significant and may reach more than 80% deviation. This wide bracketing of true thermal flow values makes the true value prediction more difficult, as illustrated in Fig. 10.

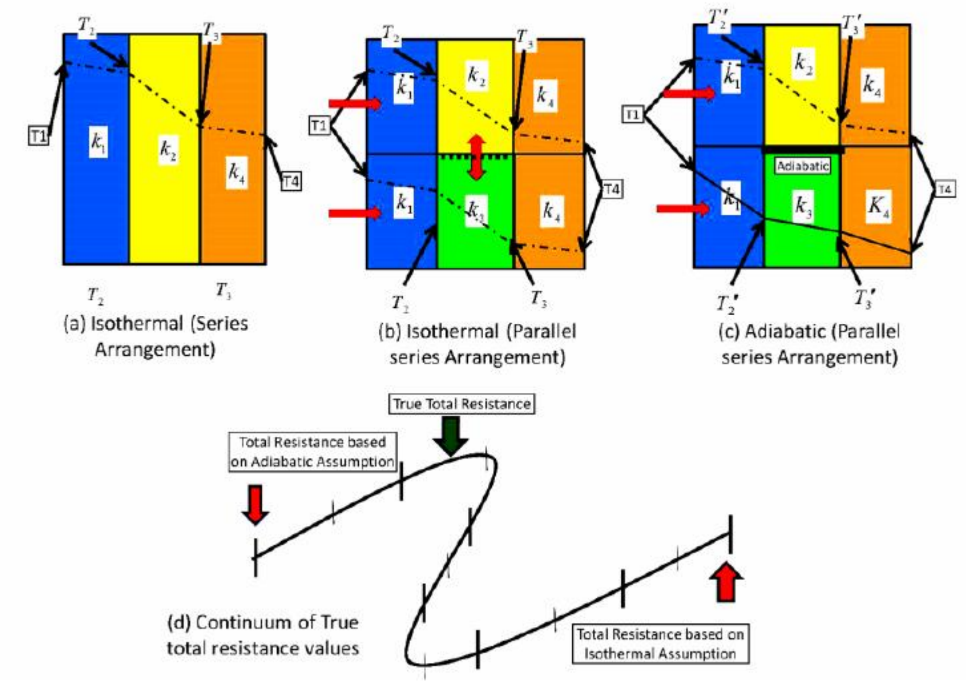

Fig. 10 Real thermal resistance values is bracketed between isothermal and adiabatic assumptions arrangements

Conclusions

Analogy is a very important cognitive device that thermal engineer should use wisely and be cautious not to be slipped in false analogy and CWTR modeling in this paper is a real practical example. Based on TTA, CWTR transparent modeling is employed to show to the thermal engineer the deep analogical concepts that lie behind isothermal and adiabatic assumptions and that these two assumptions are not equivalent but can be considered as two extreme modeling cases that bracket the true situation. The discrepancy between the heat flow calculated based on the two assumptions sours high and reaches more than 80% when there is about 100% difference in parallel k values. Several popular Heat Transfer textbooks that are mostly adopted in undergraduate settings are reviewed to find that CWTR modeling coverage needs a lot of improvement. This transparent way of presenting CWTR modeling, offered in this paper, can be smoothly included in any undergraduate heat transfer textbooks as genuine added value to CWTR modeling.

Nomenclature

A	=Area	[m^2]
l	=Wall Length	[m]
K	=Thermal Conductivity	[W.m/K]
$\dot{Q}$	=Heat Transfer Rate	[W]
R	=Thermal Resistance	[K/W]
T	=Temperature	[K]
Δx	=Wall Thickness	[m]

References

Aliedeh, M., "Tasting the Fruits" of Transparent Thinking Approach (TTA) by Developing and Validating a TTA-Based Solution Concentration Teaching-Learning Sequence (TLS): The "Kick-Off" of TTA Operationalization Phase, *JHETP*,**17**, 11-44 (2017)

Aliedeh, M.,"Building an Integrative Framework for Transparent Thinking Approach (TTA) Solution: Calling for Collaboration in Putting TTA Factory into Educational Production", *JHETP*,**18**, 103-140 (2018)

Aliedeh, M., "Call from the south for Transparent Higher Education (THE) Part 1: Transparent Thinking Approach (TTA) Core Conceptual Framework", *JHETP*,**15**, 103-130 (2015a)

Aliedeh, M., "Call from the south for transparent higher education (THE) Part 2: Extended Transparent Thinking Approach (TTA) Conceptual Framework", *JHETP*,**15**, 113-140 (2015b)

Aliedeh, M., "Call from the south for transparent higher education (THE) Part 3: Expanded Transparent Thinking Approach (TTA) Conceptual Framework and its Applications in Math, Science, and Engineering Education, *JHETP*, **15**, 121-142 (2015c)

Aliedeh, M., "Customizing Transparent Thinking Approach (TTA) by Building and Implementing TTA Toolboxes: Paving the Road for TTA Operationlization Phase", *JHETP*, **16**, 80-108 (2016)

Capizz, G., G. Sciuto, G. Cammarata, and Cammarata M. "Thermal transients simulations of a building by a dynamic model based on thermal-electrical analogy: Evaluation and implementation issue", *Appl. Ener.*, **199**, 323–334, (2017)

Cengel, Y. A. Heat Transfer, Practical Approach, 2nd Ed., McGraw Hill Company (2003)

Duit, R., Roth, W., Komorek M., and Wilbers J., "Fostering conceptual change by analogies – between Scylla and Carybdis", *Learning and Instruction*, **11**, 283-303 (2001)

Einstein, A., and Infeld L., The Evolution of Physics, Touchstone (1967)

Glynn, S., "Making science concepts meaningful to students: Teaching with analogies", In S. Mikelskis-Seifert, U. Ringelband, & M. Brückmann (Eds.), *Four decades of research in science education: From curriculum development to quality improvement*, 113- 125. Münster Waxmann (2008)

Haoa, L., Xub, F., Chenb Q., Weia M., Chenb L., and Minb Y., "A thermal-electrical analogy transient model of district heating pipelines for integrated analysis of thermal and power systems", *Appl. Therm. Eng.*, **139**, 213–221 (2018)

Harrison, A., and Treagust, D.,"Teaching and learning with analogies", In Aubusson *et al.* (Eds.). *Metaphor and Analogy in Science Education*, **11**, 11–24, Springer (2006)

Holman, J., Heat transfer, 4th Ed., McGraw Hill Book Company (1986)

Incropera, F., DeWitt, D., Bergman, T., and Lavine, A., "Fundamentals of heat and mass transfer", 6th Ed., New York, John Wiley and Sons Inc. (2002)

Jonāne, L. "Analogies in science education. Pedagogy Studies" *Pedagogika*. **119**, 116-125 (2015)

Lasance, J., "Ten Years of Boundary-Condition- Independent Compact Thermal Modeling of Electronic Parts: A Review". *Heat Tran. Eng.*, **29**, 149-168 (2008)

Ozisik, M., Heat Conduction. New York, John Wiley and Sons Inc. (1980)

Roslan, M., Azis, N., Kadir , M., Jasni, J., Ibrahim Z., and Ahmad A. "A Simplified Top-Oil Temperature Model for Transformers Based on the Pathway of Energy Transfer Concept and the Thermal-Electrical Analogy", *Energies*, **10**, 1843-1858 (2017)